

UN-Sector and Compositeness Conditions in the Bronzan-Lee Model

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The Lehmann-Symanzik-Zimmermann (LSZ) technique has been used to calculate all τ -functions of the UN-sector of the Bronzan-Lee model. Using the prescription of Liossatos, the $Z_V \rightarrow 0$ limit has been carried out for the fourier transform of the τ -functions in the sector. These limiting functions $\hat{\tau}_{\alpha, \beta}^{UN}$ are then compared with the $\hat{\tau}_{\alpha, \beta}^{UN}$ functions derived from a composite model, proposed by the foregoing author, where V is considered to be a composite particle. It has been found that when the so called composite V -particle does not appear in the initial and the final states, these τ -functions coincide. On the other hand, the limiting values of some τ -functions differ from those of the composite model, when such particles appear in the final or initial states.

I. Introduction

A considerable amount of work has been done to show the usefulness of the Lehmann-Symanzik-Zimmermann (LSZ) technique¹ in calculating relevant scattering and production amplitudes in the Lee model² and some extended Lee models. The significance of such technique was first demonstrated by Maxon and Curtis³, who have shown that the LSZ-technique can be used to calculate all processes of the V -sector. Since then Maxon⁴ used the technique to obtain the τ -functions in the $V\theta$ -sector. Scarfone⁵ extended the calculation to the VN and VV -sectors. Similar calculations have been done in the V - and U -sectors of the Bronzan-Lee model by Choudhury (to be referred as CI).

The Bronzan-Lee model, first proposed by Bronzan⁷, is an extension of the Lee-model, where one allows a nontrivial vertex function renormalization. Bronzan used a Wigner-Brillouin perturbation technique to obtain the U -propagator and $UV\theta$ vertex functions, and the relevant renormalization constants of the model. Liossatos⁸ used these results to perform the Z_V -limit in the U -sector of the Bronzan-Lee model. He derived some important Z_V -limit formulae rigorously to show how such a limiting procedure can be systematically carried out. He proposed a model which he termed the composite model of four point interaction and showed that the results of the scattering amplitudes are identical with the results obtained from the Z_V -limit technique, at least in the U -sector of the model. However, in Liossatos' model as indicated by the author, the equivalence breaks down in the $U\theta$ -sector.

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The question of equivalence of similar four point interaction models has been studied by several authors^{9–11} both in the Lee model and the Bronzan-Lee model. In all these works the authors restricted themselves to the lower sectors where the equivalence worked out well. However, one can raise the question whether the Z_V -limit can give all information a composite model can supply and vice versa. Such questions can only be answered if we study the problem of equivalence in the higher sectors of the Lee or the Bronzan-Lee model.

We choose the UN-sector of Bronzan-Lee model, where all τ -functions of relevant processes of the sector can be easily evaluated following the technique of Maxon and Curtis. The composite model proposed by Liossatos has been used to calculate the processes in the corresponding UN-sector of the model. All the processes which do not have composite initial and final states have been found to agree with the Z_V -limiting value. However in some of the τ -functions the equivalence does not hold.

In Sect. II we briefly sketch the Bronzan-Lee model. In Sect. III we give a derivation of τ -functions solving the Matthews-Salam equations in the UN-sector. In Sect. IV we carry out the $Z_V \rightarrow 0$ limit with proper precautions emphasised in Liossatos' work. In Sect. V we discuss the composite model as described by Liossatos and use the LSZ-technique to obtain solutions for corresponding τ -functions.

II. Bronzan-Lee Model

In this section we first briefly sketch the basic properties of the Bronzan-Lee model. The Hamiltonian of the model is given by (compare for the



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details CI)

$$H = H_0 + H_{\text{int}} \quad (1)$$

where *

$$H_0 = m_U Z_V \psi_U^+ \psi_U + m_V Z_V \psi_V^+ \psi_V + m_N \psi_N^+ \psi_N + \sum_k \omega a_k^+ a_k \quad (2)$$

and

$$H_{\text{int}} = g_1 Z_1 (\psi_U^+ \psi_V A + A^+ \psi_V^+ \psi_U) + g_2 (\psi_V^+ \psi_N A + A^+ \psi_N^+ \psi_V) - \delta m_U Z_U \psi_U^+ \psi_U - \delta m_V Z_V \psi_V^+ \psi_V. \quad (3)$$

A is given by the following expressions:

$$A = \sum_k X(\omega) a_k, \quad \text{where } X(\omega) = f(\omega) / \sqrt{2\omega}, \quad (4)$$

$$\omega = \sqrt{k^2 + \mu^2}.$$

The operators ψ_U , ψ_V are the renormalized annihilation operators of U and V particles. ψ_N and a_k are the annihilation operators of the N and θ particles, respectively. g_1 , g_2 are the renormalized coupling constants. δm_U and δm_V are the mass renormalization constants. Z_U , Z_V are the wave-function renormalizations of the U and V particles. Z_1 is the vertex renormalization constant.

The commutation relations are

$$[\psi_U, \psi_U^+] = 1/Z_U, \quad [\psi_V, \psi_V^+] = 1/Z_V, \quad (5)$$

$$[\psi_N, \psi_N^+] = 1 \quad [a_k, a_{k'}^+] = \delta_{kk'}.$$

It has been shown in several earlier works (CI) that the inverse of the V-particle propagator becomes

$$h^+(x) = (x - m) \alpha^+(x) \quad (6)$$

where

$$\alpha^+(x) = 1 + \frac{(x - m)}{\pi} \int_{\mu}^{\infty} d\omega \frac{\text{Im } h^+(\omega)}{(\omega - m)^2 (\omega - x - i\epsilon)} \quad (6a)$$

$$\text{Im } h^+(\omega) = \frac{g_2^2}{4\pi} f^2(\omega) \sqrt{\omega^2 - \mu^2} \theta(\omega - \mu) \quad (6b)$$

$$m = m_V - m_N. \quad (6c)$$

$B_h(z, x)$ satisfy the following important identity

$$B_h(z, x) + B_h(x - z, x) = \frac{1}{h(z) h(x - z)} - \frac{(x - 2m)}{h^+(x - m) (x - z - m) (z - m)}. \quad (14)$$

In Eq. (9) we have also introduced the following abbreviations

$$\gamma = g_1 Z_1 / g_2 \quad \text{and} \quad x_0 = m_U - m_N. \quad (15)$$

The renormalization constants in U-sectors are given by

$$Z_U = 1 + \frac{\gamma^2}{2} - \frac{\gamma^2}{2C(x, x_0)} [h^{++}(x_0 - m) - \{h^+(x_0 - m)\}^2 I_{h,x_0}^{+1}(x_0 - m)] \quad (16)$$

$$Z_U \delta m_U = Z_1^2 J_h(x_0) \quad (17)$$

$$Z_1 = C(x_0, m). \quad (18)$$

The wave function renormalization for V-particle becomes

$$Z_V = 1 - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \frac{\text{Im } h^+(\omega)}{(\omega - m)^2} \quad (7)$$

whereas the V-particle mass renormalization is

$$\delta m_V = - \frac{1}{Z_V \pi} \int_{\mu}^{\infty} d\omega \frac{\text{Im } h^+(\omega)}{\omega - m}. \quad (8)$$

The inverse of the U-particle propagator is given by

$$G(x) = Z_U (x - x_0) - \gamma^2 [J_h(x) - J_h(x_0)] \quad (9)$$

where

$$J_h(x) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \text{Im } h^+(\omega) \Phi_h^-(x, \omega) \quad (10)$$

$$\Phi_h^-(x, \omega) = \Phi_h(x, \omega - i\epsilon) \quad (10a)$$

$$\Phi_h(x, z) = \frac{1}{C(x, m)} \left\{ \frac{1}{x - z - m} + \frac{h^+(x - m)}{z - m} \right\}. \quad (10b)$$

The function $C(x, m)$ is given by

$$C(x, m) = Z_V [1 + h^+(x - m) I_{h,x}^+(x - m)]. \quad (11)$$

The functions $I_{h,x}(z)$ and $B_h(z, x)$ are given by

$$I_{h,x}(z) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \text{Im} \left[\frac{1}{h^+(\omega)} \right] \frac{1}{\alpha^+(x - \omega)} \frac{1}{\omega - z} \quad (12)$$

$$B_h(z, x) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \text{Im} \left[\frac{1}{h^+(\omega)} \right] \frac{1}{h^+(x - \omega)} \frac{1}{\omega + z - x}$$

$$= \frac{1}{x - m} [I_{h,x}(x - z) - I_{h,x}^+(x - m)]. \quad (13)$$

* Aus satztechnischen Gründen mußte im Index immer k statt \mathbf{k} gesetzt werden.

III. UN-Sector ($q_1 = 2$, $q_2 = 1$)

The sectors of the Bronzan-Lee model are designated by the eigenvalues q_1 and q_2 of the operators Q_1 and Q_2 defined by the Eq. (6) of Cl. For the UN-sector $q_1 = 2$, $q_2 = 1$. The τ -functions of this sector are defined as

$$\tau_1^{\text{UN}}(s) = \langle 0 | T[\psi_U(s) \psi_N(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (19 \text{ a})$$

$$\tau_2^{\text{UN}}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_V(s) \psi_N(s) a_k(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (19 \text{ b})$$

$$\tau_{2R}^{\text{UN}}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_U(s) \psi_N(s) a_k^+ \psi_N^+ \psi_V^+] | 0 \rangle \quad (19 \text{ c})$$

$$\tau_3^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_N(s) \psi_N(s) a_k(s) a_{k'}(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (19 \text{ d})$$

$$\tau_{3R}^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_U(s) \psi_N(s) a_k^+ a_{k'}^+ \psi_N^+ \psi_N^+] | 0 \rangle \quad (19 \text{ e})$$

$$\tau_4^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_V(s) \psi_N(s) a_k(s) a_{k'}^+ \psi_N^+ \psi_V^+] | 0 \rangle \quad (19 \text{ f})$$

$$\tau_5^{\text{UN}}(s, \omega, \omega', \omega'') = X^{-1}(\omega) X^{-1}(\omega') X^{-1}(\omega'') \langle 0 | T[\psi_N(s) \psi_N(s) a_k(s) a_{k'}(s) a_{k''}^+ \psi_N^+ \psi_V^+] | 0 \rangle \quad (19 \text{ g})$$

$$\tau_{5R}^{\text{UN}}(s, \omega, \omega', \omega'') = X^{-1}(\omega) X^{-1}(\omega') X^{-1}(\omega'') \langle 0 | T[\psi_V(s) \psi_N(s) a_k(s) a_{k'}^+ a_{k''}^+ \psi_N^+ \psi_N^+] | 0 \rangle \quad (19 \text{ h})$$

$$\tau_6^{\text{UN}}(s, \omega, \omega', \omega'', \omega''') = X^{-1}(\omega) X^{-1}(\omega') X^{-1}(\omega'') X^{-1}(\omega''') \times \langle 0 | T[\psi_N(s) \psi_N(s) a_k(s) a_{k'}(s) a_{k''}^+ a_{k'''}^+ \psi_N^+ \psi_N^+] | 0 \rangle \quad (19 \text{ i})$$

$$\tau_7^{\text{UN}}(s) = \langle 0 | T[\psi_V(s) \psi_V(s) \psi_N^+ \psi_U^+] | 0 \rangle, \quad (19 \text{ j})$$

$$\tau_{7R}^{\text{UN}}(s) = \langle 0 | T[\psi_U(s) \psi_N(s) \psi_V^+ \psi_V^+] | 0 \rangle, \quad (19 \text{ k})$$

$$\tau_8^{\text{UN}}(s) = \langle 0 | T[\psi_V(s) \psi_V(s) \psi_V^+ \psi_V^+] | 0 \rangle, \quad (19 \text{ l})$$

$$\tau_9^{\text{UN}}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_V(s) \psi_N(s) a_k(s) \psi_V^+ \psi_V^+] | 0 \rangle \quad (19 \text{ m})$$

$$\tau_{9R}^{\text{UN}}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_V(s) \psi_V(s) a_k^+ \psi_N^+ \psi_V^+] | 0 \rangle \quad (19 \text{ n})$$

$$\tau_{10}^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_N(s) \psi_N(s) a_k(s) a_{k'}(s) \psi_V^+ \psi_V^+] | 0 \rangle \quad (19 \text{ p})$$

$$\tau_{10R}^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_V(s) \psi_V(s) a_k^+ a_{k'}^+ \psi_N^+ \psi_N^+] | 0 \rangle. \quad (19 \text{ q})$$

The Matthews-Salam equations for these τ -functions are

$$Z_U \left(i \frac{d}{ds} - m_U^0 - m_N \right) \tau_1^{\text{UN}}(s) = i \delta(s) + g_1 Z_1 \sum_k X^2(\omega) \tau_2^{\text{UN}}(s, \omega) \quad (20 \text{ a})$$

$$Z_V \left(i \frac{d}{ds} - m_V^0 - m_N - \omega \right) \left\{ \begin{array}{l} \tau_2^{\text{UN}}(s, \omega) \\ \tau_{2R}^{\text{UN}}(s, \omega) \end{array} \right\} = g_1 Z_1 \tau_1^{\text{UN}}(s) + Z_V g_2 \left\{ \begin{array}{l} \tau_7^{\text{UN}}(s) \\ \tau_{7R}^{\text{UN}}(s) \end{array} \right\} + g_2 \sum_k X^2(\omega') \left\{ \begin{array}{l} \tau_3^{\text{UN}}(s, \omega, \omega') \\ \tau_{3R}^{\text{UN}}(s, \omega, \omega') \end{array} \right\} \quad (20 \text{ b})$$

$$\left(i \frac{d}{ds} - 2 m_N - \omega - \omega' \right) \left\{ \begin{array}{l} \tau_3^{\text{UN}}(s, \omega, \omega') \\ \tau_{3R}^{\text{UN}}(s, \omega, \omega') \end{array} \right\} = 2 g_2 \left\{ \begin{array}{l} \tau_2^{\text{UN}}(s, \omega) + \tau_{2R}^{\text{UN}}(s, \omega') \\ \tau_{2R}^{\text{UN}}(s, \omega) + \tau_2^{\text{UN}}(s, \omega') \end{array} \right\} \quad (20 \text{ c})$$

$$Z_V \left(i \frac{d}{ds} - m_V^0 - m_N - \omega \right) \tau_4^{\text{UN}}(s, \omega, \omega') = i \delta(s) \delta_{kk'} X^{-2}(\omega) + g_1 Z_1 \tau_{2R}^{\text{UN}}(s, \omega) + Z_V g_2 \tau_{9R}^{\text{UN}}(s, \omega) + g_2 \sum_{k''} X^2(\omega) \tau_5^{\text{UN}}(s, \omega, \omega', \omega'') \quad (20 \text{ d})$$

$$i \left(\frac{d}{ds} - 2 m_N - \omega' - \omega'' \right) \left\{ \begin{array}{l} \tau_5^{\text{UN}}(s, \omega, \omega', \omega'') \\ \tau_{5R}^{\text{UN}}(s, \omega, \omega', \omega'') \end{array} \right\} = 2 g_2 \left\{ \begin{array}{l} \tau_4^{\text{UN}}(s, \omega, \omega') + \tau_4^{\text{UN}}(s, \omega, \omega'') \\ \tau_4^{\text{UN}}(s, \omega', \omega) + \tau_4^{\text{UN}}(s, \omega'', \omega) \end{array} \right\} \quad (20 \text{ e})$$

$$\left(i \frac{d}{ds} - 2 m_N - \omega'' - \omega''' \right) \tau_6^{\text{UN}}(s, \omega, \omega', \omega'', \omega''') = 2 i \delta(s) \frac{\delta_{kk''} \delta_{k'k'''} + \delta_{kk'''} \delta_{k'k''}}{X(\omega) X(\omega') X(\omega'') X(\omega''')} + 2 g_2 [\tau_{5R}^{\text{UN}}(s, \omega'', \omega, \omega') + \tau_{5R}^{\text{UN}}(s, \omega''', \omega, \omega')] \quad (20 \text{ f})$$

$$Z_V \left(i \frac{d}{ds} - 2 m_V^0 \right) \left\{ \begin{matrix} \tau_{7N}^{UN}(s) \\ \tau_{7R}^{UN}(s) \end{matrix} \right\} = 2 g_2 \sum_k X^2(\omega) \left\{ \begin{matrix} \tau_{2N}^{UN}(s, \omega) \\ \tau_{2R}^{UN}(s, \omega) \end{matrix} \right\} \quad (20 \text{ g})$$

$$Z_V \left(i \frac{d}{ds} - 2 m_V^0 \right) \tau_{8N}^{UN}(s) = 2 i \delta(s) / Z_v + 2 g_2 \sum_k X^2(\omega) \tau_{9N}^{UN}(s, \omega) \quad (20 \text{ h})$$

$$Z_V \left(i \frac{d}{ds} - m_V^0 - m_N - \omega \right) \left\{ \begin{matrix} \tau_{9N}^{UN}(s, \omega) \\ \tau_{9R}^{UN}(s, \omega) \end{matrix} \right\} = g_1 Z_1 \left\{ \begin{matrix} \tau_{7R}^{UN}(s) \\ \tau_{7N}^{UN}(s) \end{matrix} \right\} + Z_V g_2 \tau_{8N}^{UN}(s) \\ + g_2 \sum_k X^2(\omega') \left\{ \begin{matrix} \tau_{9N}^{UN}(s, \omega) + \tau_{9N}^{UN}(s, \omega') \\ \tau_{9R}^{UN}(s, \omega) + \tau_{9R}^{UN}(s, \omega') \end{matrix} \right\} \quad (20 \text{ i})$$

$$\left(i \frac{d}{ds} - 2 m_N - \omega - \omega' \right) \left\{ \begin{matrix} \tau_{10N}^{UN}(s, \omega, \omega') \\ \tau_{10R}^{UN}(s, \omega, \omega') \end{matrix} \right\} = 2 g_2 \left\{ \begin{matrix} \tau_{9N}^{UN}(s, \omega) + \tau_{9N}^{UN}(s, \omega') \\ \tau_{9R}^{UN}(s, \omega) + \tau_{9R}^{UN}(s, \omega') \end{matrix} \right\}. \quad (20 \text{ j})$$

The solutions of the above equations are obtained by following the standard techniques of Maxon⁴, Scarfone⁵ et al.⁶. We have to convert these coupled equations in terms of the fourier transform of $\tau_\alpha^{UN}(s, \dots)$ defined by the relation

$$\tau_\alpha^{UN}(s, \dots) = \frac{i}{2\pi} \int_{-\infty}^{+\infty} dW e^{-iWs} \hat{\tau}_\alpha^{UN}(W, \dots). \quad (21)$$

Then converting these coupled equations into integral equations, we have to solve them by following the standard technique of Maxon⁴. From (20 b) and (20 c), by defining

$$\Phi_H^-(x, \omega) = \Phi_H(x, \omega - i\varepsilon) = \frac{\hat{\tau}_{2N}^{UN}(x + 2 m_N, \omega)}{g_1 Z_1 \hat{\tau}_{1N}^{UN}(x + 2 m_N) + Z_V g_2 \hat{\tau}_{7N}^{UN}(x + 2 m_N)} \quad (22)$$

and

$$W = x + 2 m_N \quad (23)$$

we obtain an integral equation

$$H(x - z) \Phi_H(x, z) = 1 - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega' \frac{\text{Im } H^+(\omega')}{\omega' + z - x} \Phi_H^-(x, \omega) \quad (24)$$

where we have set $z = \omega - i\varepsilon$. The function $H(x - z)$ is the inverse of the VN-propagator and is well known⁵. It is given by

$$H(z) = (z - x_B) D(z) \quad (25 \text{ a})$$

$$D(z) = H'(z) + \frac{(z - x_B)}{\pi} \int_{\mu}^{\infty} d\omega \frac{\text{Im } H^+(\omega)}{(\omega - x_B)^2 (\omega - z)}. \quad (25 \text{ b})$$

$H(z)$ is assumed to have a single zero for $z = x_B = W_B - 2m < \mu$, where W_B is the VN boundstate energy.

The solution of the integral Eq. (24) is^{4, 5}

$$\Phi_H(x, z) = \frac{1}{K(x, x_B)} \left\{ \frac{1}{x - z - x_B} + \frac{D(x_B) H^+(x - x_B)}{z - x_B} B_H(z, x) \right\} \quad (26)$$

where $B_H(z, x)$ is given by (13) replacing everywhere the function h by H . We also define $I_{H,x}(z)$ by replacing from (12) h by H and a by D of (25 b).

The function $B_H(z, x)$ satisfies the identity similar to (14):

$$B_H(z, x) + B_H(x - z, x) = \frac{1}{H(z)H(x - z)} - \frac{(x - 2x_B)}{D(x_B)H^+(x - x_B)(x - z - x_B)(z - x_B)}. \quad (14 \text{ a})$$

Also, the function $K(x, x_B)$ is given by

$$K(x, x_B) = Z_V [1 + D(x_B) H^+(x - x_B) I_{H,z}^+(x - x_B)]. \quad (27)$$

Since $\Phi^-(x, \omega)$ is known $\hat{\tau}_2^{\text{UN}}$ is completely determined in terms of $\hat{\tau}_1^{\text{UN}}$ and $\hat{\tau}_7^{\text{UN}}$.

From Eq. (20 i) and (20 j), defining

$$\Phi_H^-(x, \omega) = \frac{\hat{\tau}_9^{\text{UN}}(x + 2m_N, \omega)}{g_1 Z_1 \hat{\tau}_{7R}^{\text{UN}}(x + 2m_N) + Z_V g_2 \hat{\tau}_8^{\text{UN}}(x + 2m_N)} \quad (28)$$

we arrive at the same integral equation as (24).

From Eqs. (20 e) and (20 d) we get after substituting

$$\hat{\tau}_4^{\text{UN}}(x + 2m_N, \omega, \omega') = \frac{\delta_{kk'} X^{-1}(\omega)}{H^+(x - \omega')} + 2g_2^2 \frac{U^-(x, \omega, \omega')}{H^+(x - \omega)} \quad (29)$$

$$U^-(x, \omega, \omega') = U(x, \omega, \omega' - i\varepsilon) \quad (29a)$$

an integral equation

$$H(x - z)U(x, \omega, z) = P(x, \omega) + \frac{1}{x - z - \omega} - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega'' \frac{\text{Im} H^+(\omega'') U^-(x, \omega, \omega'')}{\omega'' + z - x} \quad (30)$$

where we have put

$$P(x, \omega) = \frac{H^+(x - \omega)}{2g_2^2} [g_1 Z_1 \hat{\tau}_{2R}^{\text{UN}}(x + 2m_N, \omega) + Z_V g_2 \hat{\tau}_{9R}^{\text{UN}}(x + 2m_N, \omega)] \quad (31)$$

The solution of the integral equation is known^{5, 6} and is given by

$$\begin{aligned} U(x, \omega, z) = & L(x, \omega) \Phi_H(x, z) + \frac{1}{H^+(\omega)(x - z - \omega)} \\ & + \frac{H^+(x - \omega)}{(\omega - x_B)(x - \omega - x_B)} \left[\frac{(x - z - x_B)(z - x_B)(x - 2\omega)}{(z - \omega)(x - z - \omega)} B_H(z, \omega) \right. \\ & \left. - (\omega - x_B)(x - \omega - x_B) \left\{ \frac{B_H^+(x - \omega, x)}{x - z - \omega} + \frac{B^+(\omega, x)}{z - \omega} \right\} \right] \end{aligned} \quad (32)$$

where

$$L(x, \omega) = P(x, \omega) - \frac{Z_V}{H^+(\omega)} \left[B_H^+(x - \omega, x) - B_H^+(\omega, x) - \frac{I_{H,x}^+(x - x_B)(x - 2\omega)}{(x - \omega - x_B)(\omega - x_B)} \right] \quad (32a)$$

From these results we can find out all the $\hat{\tau}$ -functions by solving some simultaneous linear equations. We get

$$\hat{\tau}_1^{\text{UN}}(W) = \frac{\Delta(W - 2m_N)}{\Delta(W - 2m_N)\Delta(W - 2m_N) - (\gamma^2/2)J_H^2(W - 2m_N)} \quad (33a)$$

$$\hat{\tau}_2^{\text{UN}}(W, \omega) = \hat{\tau}_{2R}^{\text{UN}}(W, \omega) = [g_1 Z_1 \hat{\tau}_1^{\text{UN}}(W) + Z_V \hat{\tau}_7^{\text{UN}}(W)] \Phi^-(W - 2m_N, \omega) \quad (33b)$$

$$\hat{\tau}_3^{\text{UN}}(W, \omega, \omega') = \hat{\tau}_{3R}^{\text{UN}}(W, \omega, \omega') = 2g_2 [\hat{\tau}_2^{\text{UN}}(W, \omega) + \hat{\tau}_2^{\text{UN}}(W, \omega')] \quad (33c)$$

$$\hat{\tau}_4^{\text{UN}}(W, \omega, \omega') = \frac{\delta_{kk'} X^{-2}(\omega)}{H^+(W - 2m_N - \omega')} + \frac{2g_2^2 U^-(W - 2m_N, \omega, \omega')}{H^+(W - 2m_N - \omega)} \quad (33d)$$

$$\hat{\tau}_5^{\text{UN}}(W, \omega, \omega', \omega'') = \frac{2g_2 [\hat{\tau}_4^{\text{UN}}(W, \omega, \omega') + \hat{\tau}_4^{\text{UN}}(W, \omega, \omega'')]}{W - 2m_N - \omega' - \omega'' + i\varepsilon} \quad (33e)$$

$$\hat{\tau}_{5R}^{\text{UN}}(W, \omega, \omega', \omega'') = \frac{2g_2 [\hat{\tau}_4^{\text{UN}}(W, \omega', \omega) + \hat{\tau}_4^{\text{UN}}(W, \omega'', \omega)]}{W - 2m_N - \omega' - \omega'' + i\varepsilon} \quad (33f)$$

$$\begin{aligned} \hat{\tau}_6^{\text{UN}}(W, \omega, \omega', \omega'', \omega''') = & 2 \frac{\delta_{kk''} \delta_{k'k'''} + \delta_{kk'''} \delta_{k'k''}}{X(\omega)X(\omega')X(\omega'')X(\omega''') [W - 2m_N - \omega'' - \omega''' + i\varepsilon]} \\ & + \frac{2g_2 [\hat{\tau}_{5R}^{\text{UN}}(W, \omega'', \omega, \omega') + \hat{\tau}_{5R}^{\text{UN}}(W, \omega''', \omega, \omega')]}{[W - 2m_N - \omega'' - \omega''' + i\varepsilon]} \end{aligned} \quad (33g)$$

$$\hat{\tau}_7^{\text{UN}}(W) = \hat{\tau}_{7\text{R}}^{\text{UN}}(W) = \frac{\gamma J_H(W - 2m_N)}{Z_V [A(W - 2m_N) \Delta(W - 2m_N) - (\gamma^2/2) J_H^2(W - 2m_N)]} \quad (33 \text{ h})$$

$$\hat{\tau}_8^{\text{UN}}(W) = \frac{2}{Z_V^2} \frac{\Delta(W - 2m_N)}{A(W - 2m_N) \Delta(W - 2m_N) - (\gamma^2/2) J_H^2(W - 2m_N)} \quad (33 \text{ i})$$

$$\hat{\tau}_9^{\text{UN}}(W, \omega) = \hat{\tau}_{9\text{R}}^{\text{UN}}(W, \omega) = E(W - 2m_N) \Phi^-(W - 2m_N, \omega) \quad (33 \text{ j})$$

$$\hat{\tau}_{10}^{\text{UN}}(W, \omega, \omega') = \hat{\tau}_{10\text{R}}^{\text{UN}}(W, \omega, \omega') = \frac{2g_2 [\hat{\tau}_9^{\text{UN}}(W, \omega) + \hat{\tau}_9^{\text{UN}}(W, \omega')]}{W - 2m_N - \omega - \omega' + i\varepsilon}. \quad (33 \text{ k})$$

In Eq. (33 a) $J_H(x)$ is given by the relation

$$J_H(x) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \operatorname{Im} H^+(\omega) \Phi_H^-(x, \omega). \quad (34)$$

In closed form

$$J_H(x) = \frac{1}{2Z_V K(x, x_B)} [Z_V K(x, x_B) (x - 2m + 2\delta m_V) - D(x_B) H^+(x - x_B)]. \quad (34 \text{ a})$$

The functions $A(x)$ and $\Delta(x)$ are given by

$$\begin{aligned} A(x) &= (x - 2m + 2\delta m_V - J_H(x)) \\ &= \frac{1}{2Z_V K(x, x_B)} [Z_V K(x, x_B) (x + 2\delta m_V - 2m) + D(x_B) H^+(x - x_B)]. \end{aligned} \quad (35)$$

$$\Delta(x) = Z_U(x - m_U^0 + m_N) - (\gamma^2/2) J_H(x) = Z_U(x - x_0) - (\gamma^2/2) [J_H(x) - 2J_h(x_0)]. \quad (36)$$

Finally in (33 j) we have defined

$$E(x) = \frac{2g_2}{Z_V} \frac{\Delta(x) + (\gamma^2/2) J_H(x)}{A(x) \Delta(x) - (\gamma^2/2) J_H^2(x)}. \quad (37)$$

We have thus completely determined all the τ -functions in this sector. The scattering amplitudes of all the relevant processes are related to these τ -functions as already indicated by Maxon and Curtis³. The calculations of these amplitudes are straightforward, although very laborious. We, however, refrain from quoting the final amplitudes.

IV. $Z_V \rightarrow 0$ Limit

In this section we determine the $\hat{\tau}$ -function in the $Z_V \rightarrow 0$ limit. In evaluating these limits, frequent use is made of the results obtained by Liossatos⁸ in the U-sector. We would quote some of the essential limiting formulae of some functions derived by Liossatos. We follow him in expressing the limit by a subscript l . The inverse of the V-particle propagator in the limit can be expressed as

$$h_l^+(x) = \lim_{Z_V \rightarrow 0} h^+(x) = h_l(\infty) + \frac{1}{\pi} \int_{\mu}^{\infty} \frac{d\omega' \operatorname{Im} h_l^+(\omega')}{\omega' - x - i\varepsilon} \quad (38)$$

$$h_l(\infty) = (Z_V \delta m_V)_l = - \int_{\mu}^{\infty} d\omega' \frac{\operatorname{Im} h_l^+(\omega')}{\omega' - m}. \quad (39)$$

From (7) we know

$$1 = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega' \frac{\operatorname{Im} h^+(\omega')}{(\omega' - m)^2}. \quad (40)$$

We can also put $h_l^+(x)$ in the following form

$$h_l^+(x) = (x - m) \alpha^+(x) = \frac{(x - m)}{\pi} \int_{\mu}^{\infty} d\omega' \frac{\text{Im } h_l^+(\omega')}{(\omega' - m)(\omega' - x - i\varepsilon)}. \quad (41)$$

On the other hand, the inverse VN-propagator (25 a) in the Z_V -limit can be expressed as

$$H_l^+(x) = \lim_{Z_V \rightarrow 0} H_l^+(x) = h_l(\infty) + \frac{1}{\pi} \int_{\mu}^{\infty} d\omega' \frac{\text{Im } H_l^+(\omega')}{\omega' - x - i\varepsilon}. \quad (42)$$

The function $H_l^+(x)$ can be shown to vanish at least for one particular value of x . Assuming the existence of just one bound state of VN at $x = x_B < \mu$, we can write

$$H_l^+(x) = (x - x_B) D_l^+(x) \quad (43)$$

where

$$D_l^+(x) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega' \frac{\text{Im } H_l^+(\omega')}{(\omega' - x_B)(\omega' - x - i\varepsilon)}. \quad (44)$$

Liossatos⁸ has shown the validity of an important limiting value

$$[Z_V I_{h,x}^+(x - m)]_l = 1/[2 h_l(\infty)]. \quad (45)$$

We note that $I_{hx}^+(x) = -A(x)$ in his paper. With similar arguments we can show

$$[Z_V I_{H,x}^+(x - x_B)]_l = 1/[2 h_l(\infty)]. \quad (45 a)$$

Following similar arguments for the Z_V limits of $B_h(z, x)$, we can show that $B_H(z, x)$ defined in (26) has the following limit

$$B_{Hl}(z, x) = \tilde{B}_H(z, x) + 1/2 h_l^2(\infty) \quad (46)$$

provided we fix x between μ and 2μ and z lies in the domain of analyticity of $B_H(z, x)$, that is, in the complex plane cut along the real axis from $-\infty$ to $x - \mu$. The function $B_H(z, x)$ is given by

$$\tilde{B}_H(z, x) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega' \text{Im} \left[\frac{1}{H_l^+(\omega')} \right] \frac{1}{H_l^+(x - \omega')} \frac{1}{\omega' + z - x}. \quad (47)$$

Using (45 a), (46) we can show that the function $\Phi_H(x, z)$ in (26) in the limit becomes

$$\Phi_{Hl}(x, z) = \lim_{Z_V \rightarrow 0} \Phi_H(x, z) = 2 h_l(\infty) \left[\frac{1}{(x - z - x_B) D(x_B) H^+(x - x_B)} + \tilde{B}_H(z, x) \right] + \frac{1}{h_l(\infty)}. \quad (48)$$

$\Phi_{Hl}(x, z)$ is a solution of the integral equation

$$H_l(x - z) \Phi_{Hl}(x, z) = 1 - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \frac{\text{Im } H_l^+(\omega) \Phi_{Hl}^-(x, \omega)}{\omega + z - x}. \quad (48 a)$$

Similarly, we can also show that the function $U(x, \omega', z)$ defined in Eq. (32) becomes in the limit $Z_V \rightarrow 0$

$$U_l(x, \omega', z) = \lim_{Z_V \rightarrow 0} U(x, \omega', z)$$

$$\begin{aligned} &= P_l(x, \omega') \Phi_{Hl}(x, z) + \frac{1}{H_l^+(\omega')(x - z - \omega')} + \frac{H_l^+(x - \omega')(x - 2\omega')}{(\omega' - x_B)(x - \omega' - x_B)} \left[\frac{(x - z - x_B)(z - x_B)}{(z - \omega')(x - z - \omega')} \tilde{B}_H(z, x) \right. \\ &\quad \left. - \frac{(\omega' - x_B)(x - \omega' - x_B)}{(x - 2\omega')} \left\{ \frac{\tilde{B}_H(\omega', x)}{(z - \omega')} + \frac{\tilde{B}(x - \omega', x)}{(x - z - \omega')} \right\} - \frac{\Phi_{Hl}(x, z)}{2 h_l(\infty)} + \frac{1}{2 h_l^2(\infty)} \right]. \end{aligned} \quad (49)$$

The function $U_l(x, \omega', z)$ is the solution of the integral equation

$$H_l(x - z) U_l(x, \omega', z) = P_l(x, \omega') + \frac{1}{x - z - \omega'} - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega'' \frac{\text{Im } H_l^+(\omega'')}{\omega'' + z - x} U_l^-(x, \omega'', \omega'). \quad (50)$$

We can easily verify that $U_l(x, \omega', z)$ in (49) is a solution of (50) by direct substitution and carrying out the contour integrals. $P_l(x, \omega)$ is the limit of the function defined in (31).

To get now the Z_V -limit of the τ -functions we note first that in $Z_V \rightarrow 0$

$$[Z_V J_H(x)]_l = 0. \quad (51)$$

The justification of (51) depends on the fact that in the limit $J_H(x)$ goes over to $J_{Hl}(x)$ [see Eq. (34)]

$$J_{Hl}(x) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \operatorname{Im} H_l^+(\omega) \Phi_H^-(x, \omega) \quad (52)$$

and that due to the presence of the cut-off function $J_{Hl}(x)$ can be assumed to converge, allowing us to conclude the relation (51). The limit of $K(x, x_B)$ is given by

$$[K(x, x_B)]_l = D(x_B) H_l^+(x - x_B) / [2 h_l(\omega)] \quad (53)$$

where we have used (45 a), to prove the identity (53).

We also note that from Eq. (35)

$$[Z_V A(x)]_l = 2 h_l(\infty) \quad (54)$$

and from Eq. (36)

$$[A(x)]_l = Z_{Ul}(x - x_0) - \frac{\gamma_l^2}{2} [J_{Hl}(x) - 2 J_{Hl}(x_0)] \quad (55)$$

where Z_{Ul} is the wave function renormalization constant for the U-particle derived by Liossatos⁸

$$Z_{Ul} = 1 - \frac{g_1^2}{2 g_2^2} [h_l'(x_0 - m) - h_l^2(x_0 - m) I_{h, x_0 l}^{+'}(x_0 - m)] \quad (56)$$

$$I_{h, x_0 l}^{+}(x_0 - m) = - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega' \operatorname{Im} \left[\frac{1}{h_l^+(\omega')} \right] \frac{1}{h_l^+(x_0 - \omega')} \quad (57)$$

and

$$f'(x_0 - m) = \left. \frac{df(x - m)}{dx} \right|_{x=x_0}. \quad (57 \text{ a})$$

In (55) we have also defined

$$\gamma_l = g_1 Z_{1l} / g_2 \quad (57 \text{ b})$$

where

$$Z_{1l} = h_l^+(x_0 - m) / 2 h_l(\infty) \quad (57 \text{ c})$$

and

$$J_{Hl}(x_0) = \frac{1}{\pi} \int_{\mu}^{\infty} d\omega \operatorname{Im} h_l^+(\omega) \Phi_{Hl}^+(x_0, \omega) \quad (34 \text{ a})$$

where

$$\Phi_{Hl}(x, z) = 2 h_l(\infty) \left[\frac{1}{(x - z - m) h^+(x - m)} + \tilde{B}_h(z, x) \right] + \frac{1}{h_l(\infty)} \quad (48 \text{ a})$$

is the Z_V -limiting expression of (10b), and $\tilde{B}_h(z, x)$ is obtained by substituting in (13) all h 's by h_l 's. We obtain now all $Z_V \rightarrow 0$ limit τ -functions as follows:

$$\hat{\tau}_{1l}^{\text{UN}}(W) = 1 / \Delta_l(W - 2 m_N) \quad (58 \text{ a})$$

$$\hat{\tau}_{2l}^{\text{UN}}(W, \omega) = \hat{\tau}_{2Rl}^{\text{UN}}(W, \omega) = \frac{g_1 Z_{1l}}{\Delta_l(W - 2 m_N)} \Phi_{Hl}^-(W - 2 m_N, \omega) \quad (58 \text{ b})$$

$$\hat{\tau}_{3l}^{\text{UN}}(W, \omega, \omega') = \hat{\tau}_{3Rl}^{\text{UN}}(W, \omega, \omega') = 2 g_2 [\hat{\tau}_{2l}^{\text{UN}}(W, \omega) + \hat{\tau}_{2l}^{\text{UN}}(W, \omega')] \quad (58 \text{ c})$$

$$\hat{\tau}_{4l}^{\text{UN}}(W, \omega, \omega') = \frac{\delta_{kk'} X^{-2}(\omega)}{H_l^+(W-2m_N-\omega')} + \frac{2g_2^2 U_l^-(W-2m_N, \omega, \omega')}{H_l^+(W-2m_N-\omega)} \quad (58 \text{ d})$$

$$\hat{\tau}_{7l}^{\text{UN}}(W) = \hat{\tau}_{7Rl}^{\text{UN}} = \frac{\gamma_l J_{Hl}(W-2m_N)}{2h_l(\infty) \Delta_l(W-2m_N)} \quad (58 \text{ e})$$

$$\hat{\tau}_{9l}^{\text{UN}}(W, \omega) = \hat{\tau}_{9Rl}^{\text{UN}}(W, \omega) = \frac{g_2}{h_l(\infty)} \frac{\Delta_l(W-2m_N) + (\gamma^2/2) J_{Hl}(W-2m_N)}{\Delta_l(W-2m_N)} \Phi_{Hl}^-(W-2m_N, \omega) \quad (58 \text{ f})$$

The limiting functions $\hat{\tau}_{5l}^{\text{UN}}, \hat{\tau}_{5Rl}^{\text{UN}}, \hat{\tau}_{6l}^{\text{UN}}, \hat{\tau}_{10l}^{\text{UN}}$, are obtained from (33 f), (33 g), (33 h) and (33 k) by replacing $\hat{\tau}_{\alpha}^{\text{UN}}$ by $\hat{\tau}_{\alpha l}^{\text{UN}}$ given in the equations (58 a) through (58 f).

The function $\hat{\tau}_8^{\text{UN}}(W)$ cannot be determined in the Z_V -limit. But $Z_V \hat{\tau}_8^{\text{UN}}(W)$ exists and is given by

$$[Z_V \hat{\tau}_8^{\text{UN}}(W)]_l = 1/h_l(\infty). \quad (58 \text{ g})$$

V. Composite Model

We would now like to show that the τ -functions of the corresponding processes can be obtained from the composite model suggested by Liossatos⁸, where the V-particle is considered to be a composite one. The Hamiltonian is obtained from the Hamiltonian of the Bronzan-Lee model by performing the $Z_V \rightarrow 0$ limit, which yields

$$H_c = H_{0,c} + H_{\text{int},c} \quad (59)$$

$$H_{0,c} = Z_{U1} m_U \psi_U^+ \psi_U + m_N \psi_N^+ \psi_N + \sum_k \omega(k) a_k^+ a_k \quad (59 \text{ a})$$

$$H_{\text{int},c} = \psi_{Vl}^+ \psi_{Vl} - Z_{U1} \delta m_{U1} \psi_U^+ \psi_U \quad (59 \text{ b})$$

where

$$\psi_{Vl} = (g_1 Z_{1l} \psi_U^+ A + g_2 A^+ \psi_N^+) / h_l(\infty). \quad (60)$$

Although the composite model Hamiltonian is obtained from (1) by going to the limit $Z_V \rightarrow 0$, the equivalence of the composite model with limiting results of the Bronzan-Lee model is in no way guaranteed. Liossatos did mention that in the $U\theta$ -sector such results are not the same. We now check the relevant processes in the UN-sector and show that, if we try to correlate ψ_{Vl} with the composite particle operator as done by Liossatos⁸, our results, although agreeing in some cases with those of the limiting model, differ significantly for some amplitudes.

Let us now define the τ -functions in the composite model in the UN-sector as follows:

$$\tau_{C,1}^{\text{UN}}(s) = \langle 0 | T[\psi_U(s) \psi_N(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (61 \text{ a})$$

$$\tau_{C,2}^{\text{UN}}(s, \omega) = X^{-1}(\omega) \langle 0 | T[a_k(s) \psi_N(s) \psi_{Vl}(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (61 \text{ b})$$

$$\tau_{C,2R}^{\text{UN}}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_U(s) \psi_N(s) \psi_{Vl}^+ \psi_N^+ a_k^+] | 0 \rangle \quad (61 \text{ c})$$

$$\tau_{C,3}^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_N(s) \psi_N(s) a_k(s) a_{k'}^+(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (61 \text{ d})$$

$$\tau_{C,3R}^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_U(s) \psi_N(s) a_{k'}^+ a_k^+ \psi_N^+ \psi_N^+] | 0 \rangle \quad (61 \text{ e})$$

$$\tau_{C,4}^{\text{UN}}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[a_{k'} \psi_N(s) \psi_{Vl}(s) a_k^+ \psi_N^+ \psi_{Vl}^+] | 0 \rangle \quad (61 \text{ f})$$

$$\tau_{C,5}^{\text{UN}}(s, \omega, \omega', \omega'') = X^{-1}(\omega) X^{-1}(\omega') X^{-1}(\omega'') \langle 0 | T[a_{k''}(s) a_{k'} \psi_N(s) \psi_N(s) a_k^+ \psi_N^+ \psi_{Vl}^+] | 0 \rangle \quad (61 \text{ g})$$

$$\tau_{C,5R}^{\text{UN}}(s, \omega, \omega', \omega'') = X^{-1}(\omega) X^{-1}(\omega') X^{-1}(\omega'') \langle 0 | T[a_k(s) \psi_N(s) \psi_{Vl}(s) a_{k'}^+ a_{k''}^+ \psi_N^+ \psi_N^+] | 0 \rangle \quad (61 \text{ h})$$

$$\tau_{C,6}^{\text{UN}}(s, \omega, \omega', \omega'', \omega''') = X^{-1}(\omega) X^{-1}(\omega') X^{-1}(\omega'') X^{-1}(\omega''') \times \langle 0 | T[\psi_N(s) \psi_N(s) a_{k'''}(s) a_{k''}(s) a_{k'}^+ a_k^+ \psi_N^+ \psi_N^+] | 0 \rangle \quad (61 \text{ i})$$

$$\tau_{C,7}^{\text{UN}}(s) = \langle 0 | T[\psi_{Vl}(s) \psi_{Vl}(s) \psi_N^+ \psi_U^+] | 0 \rangle \quad (61 \text{ j})$$

$$\tau_{C,7R}^{UN}(s) = \langle 0 | T[\psi_U(s) \psi_N(s) \psi_{Vl}^+ \psi_{Vl}^+] | 0 \rangle \quad (61 k)$$

$$\tau_{C,8}^{UN}(s) = \langle 0 | T[\psi_{Vl}(s) \psi_{Vl}(s) \psi_{Vl}^+ \psi_{Vl}^+] | 0 \rangle \quad (61 l)$$

$$\tau_{C,9}^{UN}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_N(s) a_k(s) \psi_{Vl}(s) \psi_{Vl}^+ \psi_{Vl}^+] | 0 \rangle \quad (61 m)$$

$$\tau_{C,9R}^{UN}(s, \omega) = X^{-1}(\omega) \langle 0 | T[\psi_{Vl}(s) \psi_{Vl}(s) a_k^+ \psi_N^+ \psi_{Vl}^+] | 0 \rangle \quad (61 n)$$

$$\tau_{C,10}^{UN}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_N(s) \psi_N(s) a_k(s) a_{k'}(s) \psi_{Vl}^+ \psi_{Vl}^+] | 0 \rangle \quad (61 p)$$

$$\tau_{C,10R}^{UN}(s, \omega, \omega') = X^{-1}(\omega) X^{-1}(\omega') \langle 0 | T[\psi_{Vl}(s) \psi_{Vl}(s) a_{k'}^+ a_k^+ \psi_N^+ \psi_N^+] | 0 \rangle. \quad (61 q)$$

Using the definition (32) we can easily verify

$$\begin{Bmatrix} \tau_{C,2}^{UN}(s, \omega) \\ \tau_{C,2R}^{UN}(s, \omega) \end{Bmatrix} = \frac{g_1 Z_{1l}}{h_l(\infty)} \tau_{C,1}^{UN}(s) + \frac{g_2}{h_l(\infty)} \sum_{k'} X^2(\omega') \begin{Bmatrix} \tau_{C,3}^{UN}(s, \omega, \omega') \\ \tau_{C,3R}^{UN}(s, \omega, \omega') \end{Bmatrix}, \quad (62 a)$$

$$\tau_{C,4}^{UN}(s, \omega, \omega') = \frac{g_1 Z_{1l}}{h_l(\infty)} \tau_{C,2R}^{UN}(s, \omega) + \frac{g_2}{h_l(\infty)} \sum_{k''} X^2(\omega'') \tau_{C,5}^{UN}(s, \omega, \omega', \omega''), \quad (62 b)$$

$$\begin{Bmatrix} \tau_{C,5}^{UN}(s, \omega, \omega', \omega'') \\ \tau_{C,5R}^{UN}(s, \omega, \omega', \omega'') \end{Bmatrix} = \frac{g_1 Z_{1l}}{h_l(\infty)} \begin{Bmatrix} \tau_{C,3}^{UN}(s, \omega', \omega'') \\ \tau_{C,3R}^{UN}(s, \omega', \omega'') \end{Bmatrix} + \frac{g_2}{h_l(\infty)} \sum_{k'''} X^2(\omega''') \begin{Bmatrix} \tau_{C,6}^{UN}(s, \omega', \omega'', \omega, \omega''') \\ \tau_{C,6R}^{UN}(s, \omega, \omega''', \omega', \omega'') \end{Bmatrix}, \quad (62 c)$$

$$\begin{Bmatrix} \tau_{C,7}^{UN}(s) \\ \tau_{C,7R}^{UN}(s) \end{Bmatrix} = \frac{g_2}{h_l(\infty)} \sum_k X^2(\omega) \tau_{C,2}^{UN}(s, \omega), \quad (62 d)$$

$$\tau_{C,8}^{UN}(s) = \frac{g_2}{h_l(\infty)} \sum_k X^2(\omega) \tau_{C,9}^{UN}(s, \omega), \quad (62 e)$$

$$\begin{Bmatrix} \tau_{C,9}^{UN}(s, \omega) \\ \tau_{C,9R}^{UN}(s, \omega) \end{Bmatrix} = \frac{g_1 Z_{1l}}{h_l(\infty)} \begin{Bmatrix} \tau_{C,7R}^{UN}(s) \\ \tau_{C,7}^{UN}(s) \end{Bmatrix} + \frac{g_2}{h_l(\infty)} \sum_{k'} X^2(\omega') \begin{Bmatrix} \tau_{C,10}^{UN}(s, \omega, \omega') \\ \tau_{C,10R}^{UN}(s, \omega, \omega') \end{Bmatrix}. \quad (62 f)$$

For the remaining τ -functions defined by (61 a) through (61 q) we can obtain the Matthews-Salam equations easily, yielding

$$Z_{Ul} \left(i \frac{d}{ds} - m_{Vl}^0 - m_N \right) \tau_{C,1}^{UN}(s) = i \delta(s) + g_1 Z_{1l} \sum_k X^2(\omega) \tau_{C,2}^{UN}(s, \omega), \quad (63 a)$$

$$\left(i \frac{d}{ds} - 2 m_N - \omega - \omega' \right) \begin{Bmatrix} \tau_{C,3}^{UN}(s, \omega, \omega') \\ \tau_{C,3R}^{UN}(s, \omega, \omega') \end{Bmatrix} = 2 g_2 \begin{Bmatrix} \tau_{C,2}^{UN}(s, \omega) + \tau_{C,2}^{UN}(s, \omega') \\ \tau_{C,2R}^{UN}(s, \omega) + \tau_{C,2R}^{UN}(s, \omega') \end{Bmatrix}, \quad (63 b)$$

$$\begin{aligned} \left(i \frac{d}{ds} - 2 m_N - \omega'' - \omega''' \right) \tau_{C,6}^{UN}(s, \omega, \omega', \omega'', \omega''') &= 2 i \delta(s) \frac{\delta_{kk''} \delta_{k'k'''} + \delta_{kk'''} \delta_{k'k''}}{X(\omega) X(\omega') X(\omega'') X(\omega''')} \\ &+ 2 g_2 [\tau_{C,5R}^{UN}(s, \omega'', \omega, \omega') + \tau_{C,5R}^{UN}(s, \omega''', \omega, \omega')], \end{aligned} \quad (63 c)$$

$$\left(i \frac{d}{ds} - 2 m_N - \omega - \omega' \right) \begin{Bmatrix} \tau_{C,10}^{UN}(s, \omega, \omega') \\ \tau_{C,10R}^{UN}(s, \omega, \omega') \end{Bmatrix} = 2 g_2 \begin{Bmatrix} \tau_{C,9}^{UN}(s, \omega) + \tau_{C,9}^{UN}(s, \omega') \\ \tau_{C,9R}^{UN}(s, \omega) + \tau_{C,9R}^{UN}(s, \omega') \end{Bmatrix}. \quad (63 d)$$

The solution of the above set of coupled equations can be obtained by a lengthy procedure. We here outline the method to determine a particular function $\tau_{C,5R}^{UN}(s, \omega, \omega', \omega'')$, which is more involved. As usual we perform the fourier transformation of the Eqs. (62 a) through (62 f) and (63 a) through (63 d) by using Equation (21). From the fourier transformation Eqs. of (62 c) and (63 c), we obtain

$$\begin{aligned} H_l^+(x - \omega) \hat{\tau}_{C,5R}^{UN}(x + 2 m_N, \omega, \omega', \omega'') &= g_{1l} Z_{1l} \hat{\tau}_{C,3R}^{UN}(x + 2 m_N, \omega', \omega'') \\ &+ 2 g_2 \left[\frac{\delta_{kk'} X^{-2}(\omega')}{x - \omega - \omega'' + i \varepsilon} + \frac{\delta_{kk''} X^{-2}(\omega'')}{x - \omega - \omega' + i \varepsilon} \right] - 2 g_2^2 \sum_{k'''} \frac{X^2(\omega''') \tau_{C,5R}^{UN}(x + 2 m_N, \omega''', \omega', \omega'')}{\omega''' + \omega - x - i \varepsilon} \end{aligned} \quad (64)$$

where $H_l^+(x)$ is given by (43). Defining

$$\begin{aligned} \hat{\tau}_{C,5R}^{UN}(x + 2 m_N, \omega, \omega', \omega'') &= \sum_{i=1,2} \left[\frac{2 g_2}{H_l^+(x - \omega)} \frac{\delta_{kk} i X^{-2}(\omega)}{H_l^+(x - \omega^i) (x - \omega' - \omega'' + i \varepsilon)} \right. \\ &\quad \left. + \frac{2 g_2^2 U_c^-(x, \omega^i, \omega)}{H_l^+(x - \omega^i) (x - \omega' - \omega'' + i \varepsilon)} \right] \end{aligned} \quad (65)$$

and remembering that $\tau_{C,3R}^{\text{UN}}$ satisfies (63 b), if we require $U_c^-(x, \omega', \omega) = U_c(x, \omega', \omega - i\epsilon)$ to satisfy

$$H(x-z)U_c(x, \omega', z) = L(x, \omega') + \frac{1}{x-z-\omega'} - \frac{1}{\pi} \int_{\mu}^{\infty} d\omega''' \frac{\text{Im } H_l^+(\omega''') U_c^-(x, \omega', \omega''')}{\omega''' + z - x} \quad (66)$$

where

$$L_c(x, \omega') = H^+(x - \omega') \cdot \frac{g_1 Z_{1l}}{2 g_2^2} \hat{\tau}_{C,2R}^{\text{UN}}(x, \omega'). \quad (67)$$

Equation (64) splits up into two identical integral equations, one of which is (66) and the other is obtained by replacing ω' by ω'' . The solution of (66) is given by $U_c(x, \omega', z) = U_l(x, \omega', z)$ defined by Equation (49). As we have mentioned earlier the solution can be verified by direct substitution.

We now quote the final results of all $\hat{\tau}_l^{\text{UN}}$'s computed by different methods. They are:

$$\hat{\tau}_{C,1}^{\text{UN}}(W) = \hat{\tau}_{1l}^{\text{UN}}(W), \quad (68 a)$$

$$\hat{\tau}_{C,2}^{\text{UN}}(W, \omega) = \hat{\tau}_{C,2R}^{\text{UN}}(W, \omega) = \hat{\tau}_{2l}^{\text{UN}}(W, \omega), \quad (68 b)$$

$$\hat{\tau}_{C,3}^{\text{UN}}(W, \omega, \omega') = \hat{\tau}_{C,3R}^{\text{UN}}(W, \omega, \omega') = \hat{\tau}_{3l}^{\text{UN}}(W, \omega, \omega'), \quad (68 c)$$

$$\hat{\tau}_{C,5}^{\text{UN}}(W, \omega, \omega', \omega'') = \hat{\tau}_{5l}^{\text{UN}}(W, \omega, \omega', \omega''); \quad \hat{\tau}_{C,5R}^{\text{UN}}(W, \omega, \omega', \omega'') = \hat{\tau}_{5Rl}^{\text{UN}}(W, \omega, \omega', \omega''), \quad (68 d)$$

$$\hat{\tau}_{C,6}^{\text{UN}}(W, \omega, \omega', \omega'', \omega''') = \hat{\tau}_{6l}^{\text{UN}}(W, \omega, \omega', \omega'', \omega'''), \quad (68 e)$$

$$\hat{\tau}_{C,4}^{\text{UN}}(W, \omega, \omega') = \frac{g_1 Z_{1l}}{h_l(\infty)} \hat{\tau}_{2l}^{\text{UN}}(W, \omega) + \frac{g_2}{h_l(\infty)} \sum_{k''} X^2(\omega'') \hat{\tau}_{5l}^{\text{UN}}(W, \omega, \omega', \omega''), \quad (68 f)$$

$$\hat{\tau}_{C,7}^{\text{UN}}(W) = \hat{\tau}_{C,7R}^{\text{UN}}(W) = \hat{\tau}_{7l}^{\text{UN}}(W), \quad (68 g)$$

$$\hat{\tau}_{C,8}^{\text{UN}}(W) = \frac{g_2}{h_l(\infty)} \sum_k X^2(\omega) \hat{\tau}_{C,9}^{\text{UN}}(W, \omega), \quad (68 h)$$

$$\hat{\tau}_{C,9}^{\text{UN}}(W, \omega) = \hat{\tau}_{C,9R}^{\text{UN}}(W, \omega) = \frac{g_1 Z_{1l}(\gamma_l/2) J_{Hl}(W - 2 m_N)}{h_l(\infty) \Delta_l(W - 2 m_N)} \Phi_{Hl}^-(W - 2 m_N, \omega), \quad (68 i)$$

$$\hat{\tau}_{C,10}^{\text{UN}}(W, \omega, \omega') = \hat{\tau}_{C,10R}^{\text{UN}}(W, \omega, \omega') = \frac{2 g_2 \{ \hat{\tau}_{C,9}^{\text{UN}}(W, \omega) + \hat{\tau}_{C,9}^{\text{UN}}(W, \omega') \}}{W - 2 m_N - \omega - \omega' + i\epsilon}. \quad (68 j)$$

We see that $\hat{\tau}_{C,9}^{\text{UN}}(W, \omega) \neq \hat{\tau}_{9l}^{\text{UN}}(W, \omega)$. As a consequence $\hat{\tau}_{C,10}^{\text{UN}}(W, \omega, \omega')$ is different from $\hat{\tau}_{10l}^{\text{UN}}(W, \omega, \omega')$. In composite model $\hat{\tau}_{C,8}^{\text{UN}}(W)$ has finite value, whereas in the Z_V -limit $\hat{\tau}_{8l}^{\text{UN}}(W)$ does not exist. We have not verified whether $\hat{\tau}_{C,4}^{\text{UN}}$ and $\hat{\tau}_{4l}^{\text{UN}}$ are exactly equal. The verification depends on the successful evaluation of a complicated contour integration, whereas the consequence is not of much significance. All other τ -functions are the same both in the Z_V -limit and the composite model.

VI. Concluding Remarks

We have shown that in the UN-sector of the Bronzan-Lee model, all τ -functions corresponding to relevant processes can be calculated by using the LSZ-formalism, which is often used by different authors in different versions of the Lee model. Following Liossatos, we have taken Z_V -limiting values of the functions and have shown that except for $\hat{\tau}_{8l}^{\text{UN}}$ all limits exist. However, if we calculate them in the "equivalent" composite model, not all of them coincide with the Z_V -limiting values. It should be observed that we used ψ_{Vl} as a kind of limiting

field of the composite V-particle in the model and constructed the τ -functions in terms of that ψ_{Vl} . This ψ_{Vl} has a very unfortunate property, that is its commutator is in the Z_V -limit theory,

$$[\psi_{Vl}, \psi_{Vl}^+] = \lim_{Z_V \rightarrow 0} 1/Z_V = \infty. \quad (69)$$

In contrast, the commutator of ψ_{Vl} in the composite model is not a c-number and contains co-efficients which are finite due to the presence of a cut off function. This difference might be one of the significant reasons why we get different results in apparently equivalent models.

We like to mention that in those processes, where ψ_{VI} does not appear in τ -functions, the results in the composite model are exactly the same as the Z_V limiting values of the Bronzan-Lee model. This may indicate that this limiting procedure can be regarded as "sensible" if we do not involve composite particles in the final or initial states. For calculating processes, where composite particle transitions take place one should rather trust a composite particle

model than the Z_V -limiting results of a model where the corresponding particle is considered as a separate entity.

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